UDC 533.6

PLAIN SELF-SIMILAR MOTIONS OF GAS HEATED BY RADIATION IN THE PRESENCE OF STRONG RE-EMISSION

PMM Vol. 37, №2, 1973, pp. 236-242

V. I. BERGEL'SON and I. V. NEMCHINOV
(Moscow)

(Received January 7, 1972)

The plane problem of interaction between the surface of a substance and a powerful incident radiation stream is considered. It is assumed that the length of the incident external radiation path is of the order of the thickness of the heated substance vapor layer and the re-emission is three-dimensional. When the radiation of the substance itself is considerable, the energy equation can be substantially simplified and in some cases reduced to a simple algebraic relationship between parameters. Self-similar solutions are derived and some numerical results of these presented.

1. Let a powerful stream of monochromatic radiation in the optical range impinge on the surface of a solid body, causing evaporation of its substance and heating the layer of its vapor to a high temperature. If the latter is considerably higher than the phase transition temperature T_v and the specific inner energy e substantially exceeds the specific heat of evaporation Q_v, it can be assumed that $T_v = Q_v = 0$. Owing to the dispersion of vapor, a reaction force acts on the solid body surface, creating a fairly high pressure p_1 (which in our formulation of the problem is an unknown parameter) in the body itself. The dispersion velocities u of the hot layer are considerably higher than the velocities u_1 of motion in the cold dense substance subjected to the effect of the reaction force (from the energy release zone in the direction of the radiation stream). Density ρ in the hot layer is considerably reduced in comparison with the solid body density ρ_1, hence the specific volume $v \gg v_1$.

On these assumptions the problem can be stated as follows: the radiation impinges on a gas which initially is infinitely dense, absolutely cold, and immobile, i.e. $v_1 = e_1 = u_1 = 0$. This formulation has been already used in solving the problem of the effect of powerful streams of monochromatic radiation [1 – 3] in which only the radiation from the source was taken into account.

In the present paper the effect of re-emission by the highly heated vapor layer is taken into consideration. It is assumed that the heat losses due to own radiation are purely three-dimensional, which is obvious in the case of a completely ionized gas [4, 5]. The expressions for specific volume luminescence and the coefficient of mass radiation absorption are of the form

$$f = f^\circ(e)\,\rho^{\beta_1}, \quad \varkappa_0 = \varkappa^\circ(e)\,\rho^{\beta_2} \tag{1.1}$$

where β_1 and β_2 are constants, and $f^\circ(e)$ and $\varkappa^\circ(e)$ are given functions of e. Note that in the region of total ionization

$$\beta_1 = \beta_2 = \beta \tag{1.2}$$

and $\beta = 1$.

For an incompletely ionized gas, particularly in the region of multiple ionization, it is possible to approximate functions f and $\varkappa_0$ by formulas (1.1) in which case β_1 and β_2 are close to each other. To simplify the subsequent analysis we assume that condition (1.2) is always satisfied.

The effect of re-emission is twofold: a part of the energy stream emitted by the layer of hot gas leaves the body, while another part is directed toward its surface (in our formulation of the problem toward the cold and dense layers of gas below the "hot zone"). Absorption of this energy generates heat and motion in layers under the zone of intensive re-emission.

Below we consider parameters only in the hot region on the assumption that it is possible to neglect the motion and heating of vapors below the zone of intensive re-emission.

2. The system of equations of motion, continuity, energy, incident radiation transport, and of state for a one-dimensional plane motion of gas heated by radiation, with allowance for three-dimensional emission of radiation of the continuous spectrum, is

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \qquad \frac{\partial v}{\partial t} - \frac{\partial u}{\partial m} = 0$$
$$\frac{\partial e}{\partial t} + p\frac{\partial v}{\partial t} = -f - \frac{\partial q}{\partial m}, \qquad \frac{\partial q}{\partial m} = -\varkappa_0 q, \qquad e = \frac{pv}{\gamma - 1} \tag{2.1}$$

where m is the Lagrangian mass coordinate taken from the boundary between gas and vacuum, t is the time, q is the density of the incident monochromatic radiation stream, and γ is effective adiabatic exponent. Functions f and $\varkappa_0$ are defined by formulas(1.1).

In conformity with the formulation of the problem given in Sect. 1 the initial and boundary conditions are defined by

$$u(m, 0) = v(m, 0) = e(m, 0) = 0 \tag{2.2}$$
$$q(0, t) = q_0(t), \quad p(0, t) = 0 \tag{2.3}$$

The fundamental premise of the subsequent analysis is the assumption that in the case of substantial re-emission the internal energy and work produced by the expansion of gas can be neglected, i.e. it is possible to neglect the left-hand side of the equation, which is small in comparison with the two terms in its right-hand side (validity of this assumption will be proved below).

As the result, the equation of energy in system (2.1) is replaced (with allowance for the equation of transport) by a certain expression of the form

$$f = \varkappa_0 q \quad \text{or} \quad q = \frac{f^\circ(e)}{\varkappa^\circ(e)} \equiv \varphi(e) \tag{2.4}$$

which defines the relation between q and e.

3. Let us consider the case of gas heated by a stream of monochromatic radiation of constant density $q_0(t) \equiv q_0 = \text{const}$. It then follows from formula (2.4) that the inner energy e_0 at the boundary of gas and vacuum is also constant. We take this energy as typical for our problem. We point out that in this case $\varphi(e)$ is an arbitrary function of e which, for instance, can be specified in a tabulated form (the only requirement imposed on $\varphi(e)$ is that it must be monotonic within the range of variation of characteristic parameters in every specific variant of the problem). The effective adiabatic exponent γ in this case, can also be considered as a given function of e, however, for simplicity of calculations we shall assume $\gamma = \text{const}$.

We introduce self-similar variables defined by formulas

$$\begin{gathered}
p\,(m,\,t) = (\gamma - 1)^{1-s/2} e_0^{\,1-s/2}\, A^{-s} t^{-s} P\,(\mu) \\
v\,(m,\,t) = (\gamma - 1)^{s/2} e_0^{\,s/2} A^{s} t^{s} V\,(\mu) \\
u\,(m,\,t) = (\gamma - 1)^{1/2}\, e_0^{\,1/2}\, U\,(\mu) \\
e\,(m,\,t) = e_0 E\,(\mu), \quad q\,(m,\,t) = q_0 Q\,(\mu) \\
m = (\gamma - 1)^{\beta s/2} e_0^{\,\beta s/2}\, A^{-s} t^{\beta s} \mu, \quad s = 1\,/\,(\beta + 1), \quad A = \varkappa^{\circ}\,(e_0)
\end{gathered} \tag{3.1}$$

For a completely ionized gas $\beta = 1$, and from (3.1) we obtain the following laws of variation with time of pressure, density, and the characteristic quantity of the heated mass:

$$p \sim t^{-1/2}, \quad \rho \sim t^{-1/2}, \quad m \sim t^{1/2}$$

The characteristic quantities of inner energy e (and temperature T) and of dispersion velocity u do not vary with time. Let us compare these formulas with the solution of a similar problem [1, 2]

$$p \sim t^{-1/8}, \quad \rho \sim t^{-3/8}, \quad m \sim t^{3/4}, \quad e \sim t^{1/4}, \quad u \sim t^{1/8}$$

in which no allowance is made for re-emission.

At the instant at which the effect of re-emission becomes considerable $(t = t_r)$ the characteristic parameters determined in [1, 2] and those of the self-similar solution considered here are close to each other.

Comparing the time-dependence of parameters obtained here, we come to the conclusion that in both cases the characteristic dispersion velocities are close to each other even after a very considerable time from the beginning of intensive re-emission $(t \gg t_r)$. Temperatures and densities differ only slightly, while pressures differ (by, say, one order) only for times which considerably exceed (by, say, two orders) the time elapsed between the commencement of radiation and the beginning of intensive re-emission. Re-emission of virtually the whole supplied energy begins at the same time. In fact, during the period of time exceeding by two orders t_r, the kinetic energy and the inner energy of a unit mass remain unchanged, while the mass increases by one order, which means that the total energy represents approximately $1/_{10}$-th of supplied energy. The momentum created by the dispersion of the vapor layer also increases by one order, owing to the increase of mass. Thus, in spite of intensive re-emission, the dispersing vapors continue to exert a mechanical action, albeit a somewhat weaker one, on the deeper layers of the substance. (We recall that the effects related to the motion below the "hot zone" induced by absorption of radiation energy are not considered here).

We introduce the notation

$$F = \frac{f^{\circ}}{f^{\circ}\,(e_0)}, \qquad K = \frac{\varkappa^{\circ}}{A}, \qquad \Phi = \frac{\varphi}{\varphi\,(e_0)}$$

Using this notation, from (2.1) we obtain the following system of ordinary differential equations:

$$\begin{gathered}
\frac{\beta}{\beta + 1}\,\mu U' = P' \\
\frac{P}{\mu}\,P'\left[\frac{1}{E} - \frac{\mu^2}{P^2}\left(\frac{\beta}{\beta+1}\right)^2\right] = \frac{\beta}{(\beta+1)^2} - \left(\frac{\beta}{\beta+1}\right)^2 \frac{\mu}{E}\,E' \\
\frac{d}{dE}\,\{\ln\,[\Phi\,(E)]\}\,E' = -\,K\,(E)\,E^{-\beta} P^{\beta}
\end{gathered} \tag{3.2}$$

$$E = PV, \qquad Q = \Phi(E) = F(E)/K(E)$$

where the prime indicates differentiation with respect to μ. The initial and boundary conditions (2.2) and (2.3) are now represented by the following system of boundary condition:

$$P = 0, \ E = 1 \quad \text{for} \quad \mu = 0$$
$$V = E = U = 0 \quad \text{for} \quad \mu = \mu_1 \qquad (3.3)$$

For any a priory specified functions $F(E)$ and $K(E)$ the system (3.2),(3.3) can be numerically integrated.

4. Let the dependence of the density of incident radiation stream on time be defined by the power law

$$q_0(t) = q^\circ t^n, \quad n \geqslant 0 \qquad (4.1)$$

and let $f^\circ(e)$ and $\varkappa^\circ(e)$ be power functions of their arguments

$$f^\circ(e) = k_f e^\delta, \quad \varkappa^\circ(e) = k_\varkappa e^{-\alpha} \qquad (4.2)$$

where q°, $k_\varkappa$ and k_f are dimensional constants. Then $\varphi(e)$ in (2.4) is also a power function of e, and from (2.4) we obtain for the characteristic quantity e_0 the following expression in terms of power function of t

$$e_0 = k_e t^{n/\omega}, \qquad \omega = \delta + \alpha, \ k_e = (q^\circ k_\varkappa / k_f)^{1/\omega} \qquad (4.3)$$

For a completely ionized gas we have: $\alpha = {}^3/_2$, $\delta = {}^1/_2$, $\omega = 2$, $k_\varkappa \sim \varepsilon_0^{-2}$, $k_e \sim \varepsilon_0^{-1}$, where ε_0 is the energy of monochromatic radiation quanta.

Introducing self-similar variables by formulas

$$p(m, t) = (\gamma - 1)^{1-s/2} k_\varkappa^{-s} k_e^{s(\beta+\alpha+1/2)} t^g P(\mu)$$
$$v(m, t) = (\gamma - 1)^{s/2} k_\varkappa^{s} k_e^{s(1/2-\alpha)} t^d V(\mu)$$
$$u(m, t) = (\gamma - 1)^{1/2} k_e^{1/2} t^{1/2\, n/\omega} U(\mu) \qquad (4.4)$$
$$e(m, t) = k_e t^{n/\omega} E(\mu), \quad q(m, t) = q^\circ t^n Q(\mu)$$
$$m = (\gamma - 1)^{\beta s/2} k_\varkappa^{-s} k_e^{s(\alpha+\beta/2)} t^c \mu$$
$$s = 1/(\beta + 1), \qquad g = s\left[-1 + \frac{n}{\omega}(\beta + \alpha + {}^1/_2)\right]$$
$$d = s\left[1 + \frac{n}{\omega}({}^1/_2 - \alpha)\right], \quad c = s\left[\beta + \frac{n}{\omega}(\alpha + \beta/2)\right]$$

we obtain the following system of self-similar equations:

$$\frac{n}{2\omega} U - c\mu U' + P' = 0$$
$$\frac{P}{\mu} P' \left(\frac{1}{E} - \frac{\mu^2}{P^2} c^2\right) = cd + \frac{c^2}{\omega} \mu E^{-(\alpha+\beta)} P^\beta - \frac{n}{2\omega} \frac{P}{\mu} \frac{U}{E}$$
$$E' = -\frac{1}{\omega} E^{1-\alpha-\beta} P^\beta, \qquad E = PV, \qquad Q = E^\omega \qquad (4.5)$$

Boundary conditions (3.3) remain unchanged.

The looked for solution obviously corresponds to a heating wave spreading over the background with zero values of U, V and E and generating ahead of itself, similarly to a piston, a region of increased pressure $P = P_1$ up to $\mu = \infty$, where the shock-wave front which converts the state of gas from $U = V = P = 0$ to $U = V = 0$ and $P = P_1 = \text{const}$ is present. It can be shown that the assumption (2.4) is asymptotically satisfied.

In fact, for each fixed μ the terms $\partial e/\partial t$ and $p\partial v/\partial t$ vary with time as $t^{n/\omega-1}$ and the terms f and $\partial q/\partial m$ as t^{n-c}, and for sufficiently great t the first of these are small in comparison with the second, if condition

$$n/\omega\,[(\beta+1)(1-\omega)+\alpha+\beta/2]<1 \tag{4.6}$$

is satisfied.

For $n = 0$ (constant stream) inequality (4.6) is satisfied for any α, β and ω. In the particular case of a completely ionized gas the expression in brackets in (4.6) vanishes, hence the inequality (4.6) is satisfied for any n.

Owing to the considerable number of possible combinations of parameters α, β and ω, these are not considered here. We stress, however, that for every set of α, β and ω the admissible values of n are determined by the condition of fulfilment of inequality (4.6).

Inequality (4.6) was derived on the assumption that the proportionality coefficients in the considered power formulas (the dimensionless functions of self-similar variable μ) are of the order of unity. Since at point $\mu = 0$ (the boundary with vacuum) $f = \varkappa_0 = 0$, the assumption (2.4) is, evidently, not satisfied there.

Points $\mu = 0$ and $\mu = \mu_1$ are singular points of system (4.5). In the right neighborhood of point $\mu = 0$ we have the following laws of variation of functions (correct to within magnitudes of higher order of smallness):

$$P = \left(\frac{\beta}{\beta+1}\right)^{1/2}\mu, \qquad U = \left(\frac{\beta+1}{\beta}\right)^{1/2}\ln\mu + U_0 \qquad (n=0) \tag{4.7}$$

$$P = D\mu\,(-\ln(C_2\mu))^{1/2}, \qquad U = -\frac{2\omega}{n}D(-\ln(C_2\mu)) \qquad (n>0) \tag{4.8}$$

$$D = \left[\frac{n}{\omega}\left(2+\frac{n}{2\omega}\right)\right]^{1/2}$$

where U_0 and $C_2 > 0$ are arbitrary constants. In this case E can be calculated by formula

$$E = \left(1 - \frac{\alpha+\beta}{\omega}\int_0^\mu P^\beta\,d\mu\right)^{1/(\alpha+\beta)} \tag{4.9}$$

Note that the gas discharge velocity into vacuum is infinite, as in the case of isothermal dispersion.

In the left neighborhood of $\mu = \mu_1$ we have

$$E = \frac{\alpha+\beta}{\omega}P_1^\beta(\mu_1-\mu)^{1/(\alpha+\beta)} \tag{4.10}$$

$$P = (P_1{}^2 - 2c^2\mu_1{}^2E)^{1/2}, \qquad U = \frac{P-P_1}{c\mu}$$

Expansion of the self-similar variables in the neighborhood of the singular point $\mu = 0$ yields

$$\frac{\partial e}{\partial t} + p\frac{\partial v}{\partial t} = C_1t^{n/\omega-1}$$

where C_1 is a constant and f and $\partial q/\partial m$ are, in turn, represented in the form of constants multiplied by $P^{\beta}(\mu)\, t^{n-c}$, with $P(\mu)$ increasing with μ at a rate not lower than linear.

Comparison of the derived formulas shows that the size of point $\mu = 0$ neighborhood in which the assumption of smallness of $\partial e / \partial t + p \partial v / \partial t$ is not valid, since it decreases with time, hence for any arbitrarily small $\mu \neq 0$ an instant t, after which condition (2.4) is satisfied, does always exist. Similar reasoning applies also to the proof of fulfilment of condition (2.4) in the neighborhood of point $\mu = \mu_1$ of the heating wavefront.

System (4.5) is numerically integrated from $\mu = 0$ to $\mu = \mu_1$, using in the neighborhood of $\mu = 0$ expansion (4.7) for $n = 0$ or (4.8) for $n > 0$ and selecting the independent parameter U_0 (or, respectively, C_2) so as to satisfy the condition $U = 0$ for $E = 0$.

Note that system (4.5) can, also, be numerically integrated by commencing at point $\mu = \mu_1$, using the related expansions (4.10) and selecting the independent parameter P_1 so as to satisfy the condition $P = 0$ for $\mu = 0$. The second necessary condition $E = 1$ for $\mu = 0$ can be readily satisfied, owing to the invariance of system (4.5) with respect to the linear variation of scales

$$P = C_P \bar{P}, \quad U = C_U \bar{U}, \quad \mu = C_\mu \bar{\mu}, \quad E = C_E \bar{E} \qquad (4.11)$$

$$C_U = C_E^{1/2}, \quad C_\mu = C_E^{s(\alpha+\beta/2)}, \quad C_P = C_E^{1+s(\alpha+1/2)}, \quad C_E = E(0)$$

$$s = 1 / (\beta + 1)$$

Recalculation by formulas (4.11) yields the solution of the system in the case of numerical integration from $\mu = \mu_1$ to $\mu = 0$.

The distribution of dimensionless magnitudes E, P, U and V with respect to the self-similar coordinate μ is shown in Figs. 1 and 2 for the case of a completely ionized gas.

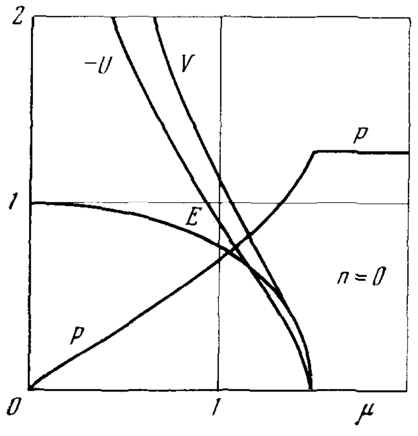

Fig. 1

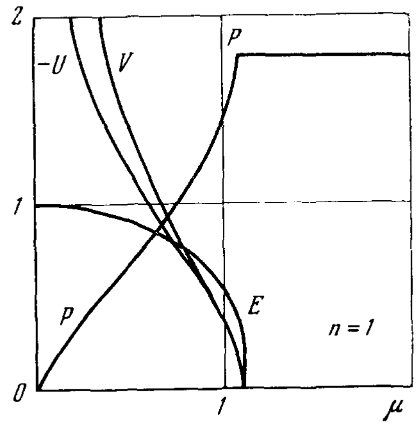

Fig. 2

Figure 1 relates to constant density radiation stream ($n = 0$) and Fig. 2 to a linearly increasing stream ($n = 1$). In these figures the heating wavefronts lie at points with coordinates $\mu_1 = 1.48$ and 1.08, respectively, and the dimensionless pressures at the front are, respectively, $P_1 = 1.33$ and 1.79 (constants $U_0 = -0.934$ and $C_2 =$

32.4). Note that a sharply defined heating wavefront (with infinite derivatives) exists only, if condition $\alpha + \beta > 1$ is satisfied. For $\alpha + \beta \leqslant 1$ the front is not sharply defined.

BIBLIOGRAPHY

1. Afanas'ev, Iu. V., Krol', V. M., Krokhin, O. N. and Nemchinov, I. V., Gasdynamic processes in heating of a substance by laser radiation. PMM Vol. 30, №6, 1966.
2. Krol', V. M., Plane self-similar motions of a heat conducting gas heated by radiation. PMTF, №4, 1968.
3. Nemchinov, I. V., Steady-state motion of radiation-heated vapors of a material in the presence of lateral spreading flow. PMM Vol. 31, №2, 1967.
4. Basov, N. G. and Krokhin, O. N., Conditions of plasma heating up by optical radiation quantum generator. ZhETF, Vol. 46, №1, 1964.
5. Dawson, J. M., On the production of plasma by giant pulse lasers. Phys. Fluids, Vol. 7, №7, 1964.

Translated by J. J. D.

UDC 532.522

ON EXTERNAL FLOWS INDUCED BY JETS IN A VISCOUS INCOMPRESSIBLE FLUID

PMM Vol. 37, №2, 1973, pp. 243-246
L. P. GORBACHEV, N. V. NIKITIN and E. P. POTANIN
(Moscow)
(Received July 6, 1972)

A method is proposed for calculating flows induced by fluid sucked into a jet outside the boundary layer region. The jet is simulated by a set of sinks whose intensity is specified in terms of known solutions of the boundary layer theory. With few exceptions [1 – 3] problems of the boundary layer theory related to jet-like flows of viscous fluids are solved for high Reynolds numbers. In such approximation the presence of suction of fluid into the jet is a distinctive feature, which has the effect of inducing motion of the fluid in the space outside the jet. Since in the external region of flow the velocities of fluid motion and the Reynolds numbers are not high, the inertia terms in the Navier-Stokes equations can be neglected. A fine jet can be simulated by a system of sinks distributed along its axis. The intensity of sinks is deremined by solutions which define jet-like flows within the limits of the boundary layer theory [4, 5]. The linear problem of external flow thus formulated can be analytically solved for various kinds of jet-like fluid motions. Several examples are presented.

1. External flow induced by a jet flowing from a narrow tube. We introduce a system of spherical coordinates with origin at the jet outlet and angle θ measured from the jet axis. We seek components of velocity and pressure in the form

$$v_r = \frac{\nu}{r} f(\theta), \quad v_\theta = \frac{\nu}{r} \varphi(\theta), \quad \frac{P}{\rho} = \frac{\nu^2}{r^2} F(\theta) \tag{1.1}$$